Is the variogram a good tool for assessing the spatial variability of vertical profiles of reflectivity?

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1 Introduction

Various methods have been proposed in the literature to correct for the errors arising from nonuniform vertical profiles of reflectivity (VPR). All these methods consist in estimating the shape of the VPR and to use it for extrapolating the radar reflectivities measured at high altitude towards ground level. These methods require an assumption on the spatial homogeneity of the VPR over appropriate subdomains during a considered time step. This assumption of spatial homogeneity must therefore ideally be verified in order to apply any correction. In this paper we will explore the ability of a geostatistical tool called variogram to assess the spatial variability of the VPR in volume reflectivity files. The data are measured by the Wideumont radar in Belgium. It is a C-band Doppler radar that performs a 5-elevation scan from 0.3° to 6.0° every 5 minutes and a 10-elevation scan from 0.5° to 17.5° every 15 minutes. Some relevant parameters of the 10-elevation scan are given in Delobbe and Holleman (2006). The final objective of this study is to characterize the spatial and temporal variability of VPRs for different types of meteorological situations and to determine consistent spatial and temporal scales for VPR identification and correction. In this paper we will concentrate on the spatial variability aspect only. This paper is organized as follows. We first develop the theory related to variograms applied on VPRs. Then an analysis of one theoretical example is given. This analysis will illustrate some difficulties in the calculation of the variograms. Finally, an analysis of the variograms obtained for several selected observed situations is done and the results are discussed.

2 Variograms of VPRs

The goal of this section is to characterize the spatial scales at which the VPR varies. In other words, we would like to know the decorrelation distance beyond which two selected VPRs may be considered as independent. A tool to answer this question is called the variogram. For scalar measurements Z and Z' of coordinates x and x' in the plane, the semivariogram g(h) at a distance h defined by

$$h = \mathrm{dist}(x,x') \tag{1}$$

is given by the expression:

$$g(h) = \frac{1}{2} E\{dist(Z,Z')^2\} \tag{2}$$

In the previous equation, *E{}* stands for mathematical expectation (mean). In equations (1) and (2) the distance *dist* is defined by the absolute difference of its two arguments. Usually, *g(h)* is an increasing function of *h*. It usually happens that the function *g* reaches a sill beyond a distance h_0. This distance is then called the decorrelation distance. Practically, classes of distances are chosen in order to approximate the expectation in equation (2). More precisely for one selected class of distances all pairs of points *x* and *x'* whose distance *h* belongs to the class are used to approximate the expectation in equation (2). In our study we do not have scalar measurements but vectorial ones. Indeed, each VPR can be viewed as a vector of several components. If we are able to define a distance *d* between two VPRs, then we can apply equation (2) in order to calculate the semivariogram between VPRs separated by a distance h:

$$g(h) = \frac{1}{2} E\{d(VPR,VPR')^2\} \tag{3}$$

We have chosen to work with the distance *d* defined as follows:

$$d = \frac{\sum_{i=1}^{n}\left([VPR(i) - VPR(i_{min})] - [VPR'(i) - VPR'(i_{min})]\right)}{n} \tag{4}$$

where *i* is a height level index, i_{min} is the index of the lowest height level for which the VPRs are both defined and *n* is the

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total number of height levels for which the VPRs are both defined i.e. when their reflectivity values are both above the 7 dBZ threshold.

In formula (4) a normalization of the profiles is done so that two normalized profiles have an identical lowest starting value. The aim is to compare the shape of the profiles rather than their absolute values. When applying formulas (3) and (4) we have chosen to express the VPRs in *dBZ* units so that the semivariogram is expressed in dBZ^2 units.

3 Theoretical example

We have performed some tests on a theoretical example where the idealized VPR is spatially constant over the entire domain. In this case, the true variogram is equal to zero. This idealized VPR corresponding to a bright band situation is depicted in Fig. 1.

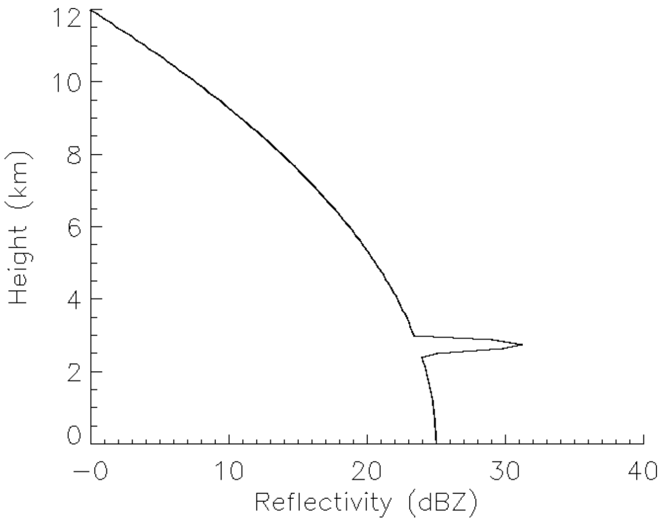

Fig. 1. Idealized VPR corresponding to a bright band situation.

The profile of reflectivity measured by the radar is not the same as the idealized profile because of the width and power distribution of the radar beam and the limited number of elevation angles scanned by the radar. We do not consider here the effect of attenuation. The profile measured by the radar is given by the convolution of the radar beam pattern with the true profile of reflectivity. We have assumed that the power distribution within the beam is gaussian. Since we know how the beam height and width increase with range in standard conditions we were able to simulate the measured VPR at the different selected ranges by calculating an appropriate convolution. Fig. 2 shows a series of apparent VPRs, 5km apart. Each VPR in this series is the result of a convolution of the radar beam pattern with the single idealized profile, the elevation angles of the 10-elevation scan of the Wideumont Radar being used. We observe that the bright band is more and more smoothed with increasing distance from the radar and that this bright band is only detected for ranges smaller than 100 km.

From the idealized profile of Fig. 1 assumed spatially constant over the entire domain, it was possible to generate an artificial measured volume file by calculating a convolution of the idealized VPR with the radar beam for ranges from 0 to 240 km. An associated variogram depicted by Fig. 3 (upper curve) was then calculated using equation (3). This variogram was calculated using 100 levels between 0 and 12 km and a cutoff threshold of 7 dBZ was used (i.e. the values under this threshold were not taken into account).

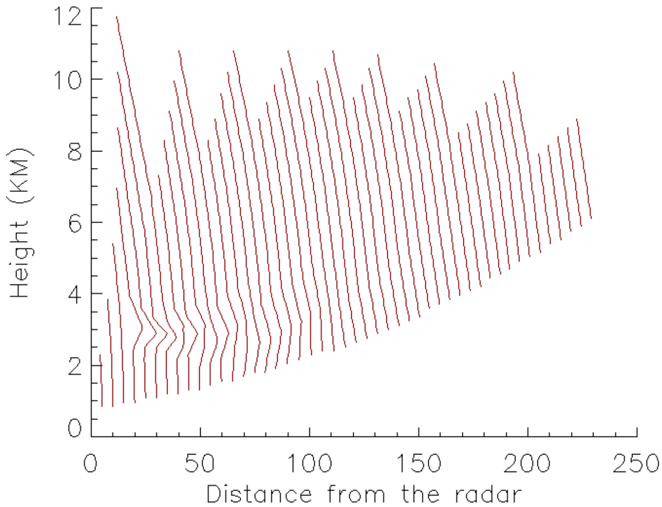

Fig. 2. Series of apparent VPRs, 5 km apart, for 10 elevations

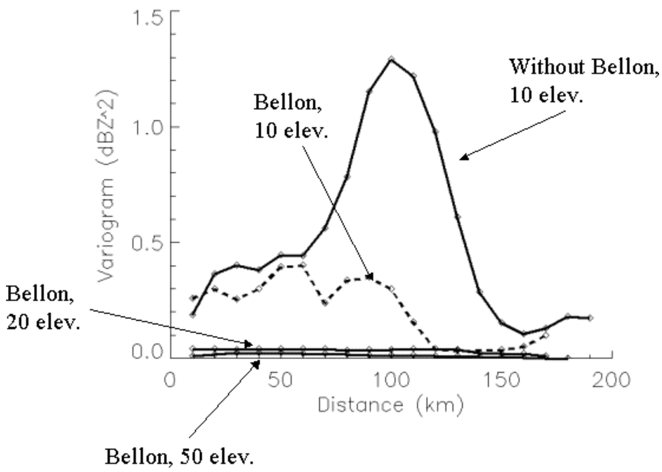

Fig. 3. Four theoretical variograms. Upper curve: with ten elevations without any Bellon's correction. Three lowest curves: with 10, 20 and 50 elevations and with a Bellon's correction.

We observe that the variogram globally increases up to the distance of 100 km (from 0.2 dBZ^2 to 1.3 dBZ^2) and then decreases to reach a value of 0.2 dBZ^2 for a distance of 190 km. The increase of the variogram can be explained by the fact that the distance *d* (given by equation (4)) between apparent VPRs increases as the geographical distance between them increases. The decrease after 100 km can also be explained: due to the increasing height of the measurements, only the upper part of the VPRs are compared, i.e. where the bright band is not detected. The variogram shown in Fig. 3 reflects the variations of the convolved VPRs. Those variations are only due to sampling errors caused by the measurement height effect and the beam broadening effect (the fact that the beam width increases with range). Theoretically if we could correct for the sampling errors, the variogram obtained should be equal to zero since the idealized VPR has been chosen constant everywhere. The question is thus: how to compare apparent VPRs situated at different distances from the radar ? A possible solution is to make the two VPRs comparable by using a convolution to be applied on the closest VPR such that the two VPRs will be similarly smoothed before comparing them. Bellon et al. (2005) in their appendix give a formula for such a convolution. If the closest VPR is situated at a range r_0 and the furthest VPR at a range r, the closest VPR should be smoothed by convolving it at a range

$$r_{eff} = \sqrt{\left(r^2 - r_0^2\right)} \qquad (5)$$

in order to be comparable to the furthest VPR.

We have applied this strategy (that we call the Bellon's correction) to calculate the variogram shown in Fig. 3 (labelled "Bellon, 10 elev."). We observe that the variogram varies between the values of 0.25 dBZ² and 0.40 dBZ² for distances lower than 100 km and that then it decreases to the values between 0.05 dBZ² and 0.10 dBZ². The values of this variogram are for most of the distances lower than the corresponding values of the upper variogram of Fig. 3. It means that for this example the Bellon's convolution effectively reduces the distances *d* between the VPR pairs which are compared. Since we are in a theoretical framework where the behavior of the radar is simulated we can also work with more than ten elevation angles. When doing so we use several angles in geometric progression between 0.5 and 17.5 degrees. If we increase the number of angles we observe that the variograms are getting closer to zero as illustrated by Fig. 3 (two lowest curves). Fig. 4 shows for a selected pair of distances that the Bellon's correction applied to the closest apparent VPR is very close to the furthest apparent VPR when we work with 50 elevation angles and that it approaches this VPR when 10 elevation angles are used.

4 Variograms obtained from the measured volume files

We have calculated variograms for 30 selected situations. We have worked with VPRs calculated on stratiform zones. The separation between convective and stratiform zones is based on the Steiner algorithm (Steiner et. al, 1995). The calculation of those variograms has been performed with and without a preliminary Bellon's correction. In order to explain in more details the calculation of the variograms let us consider two VPRs located at different distances from the radar. If these two VPRs were known, their comparison would be immediate. But these two VPRs are only measured i.e. convolved with a small number of radar beams with different elevation angles. The result of this measurement leads to a discretization of the original profiles and this in polar coordinates. These discretized profiles are then converted into cartesian coordinates by mean of an interpolation process. Since the two profiles are located at different distances from the radar they are not sampled in the same way by the radar. Optionaly, the nearest profile undergoes a transformation which we called the Bellon's correction. It is then interpolated in order to be compared with the distant VPR.

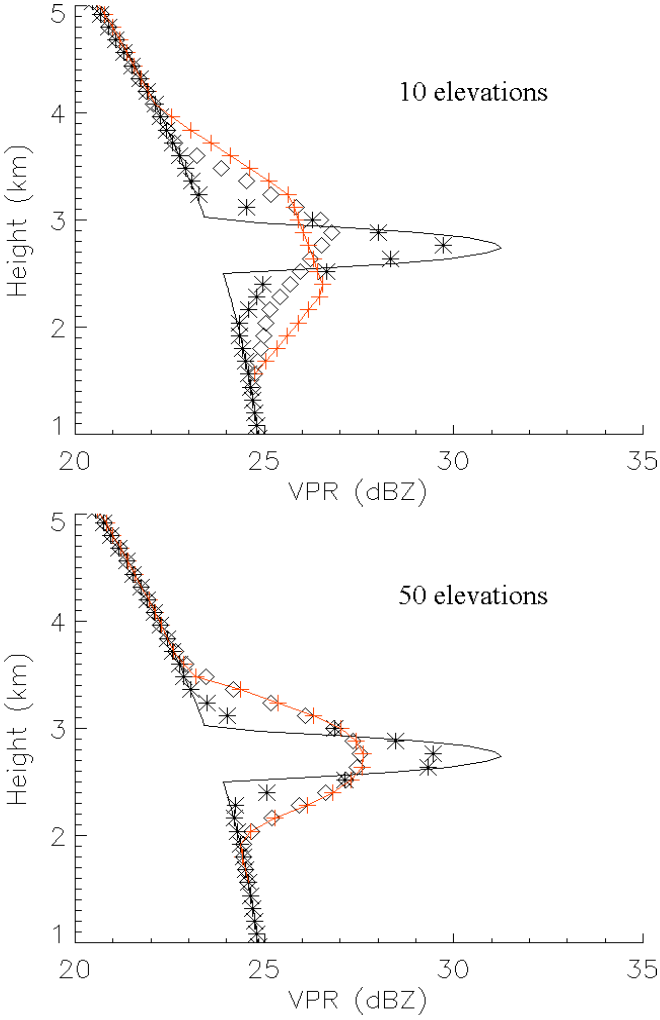

Fig. 4. Original theoretical VPR (solid curve), apparent VPR seen by the radar at a distance of 30 km (curve labeled with '*'), apparent VPR seen by the radar at a distance of 70 km (red solid curve labeled with '+') and Bellon's convolution applied to the closest apparent VPR such that it is seen at the distance of the furthest apparent VPR (diamond curve). 10 (upper figure) and 50 (lowest figure) elevation angles geometrically spaced between 0.5° and 17.5° have been used.

Fig. 5 shows examples of variograms obtained for a volume scan measured on 29/07/2005 at 00:04 UTC and calculated with (lowest continuous curve) and without (upper continuous curve) a preceding Bellon's correction. The variograms use 24x24 averaged VPRs on 24x24 10 km x 10 km squares. The studied domain is thus a square of dimensions 240 km x 240 km centered on the radar. Each point of the variograms is calculated only if a sufficient number of couples is available (threshold = 10). An exponential model (dashed line) has been fitted to each experimental variogram. This model gives the nugget variance (C0) the sill (C1) as well as the decorrelation distance (d0). This exponential model is described by the following equation:

$$g(d) = C0 + (C1 - C0)\left[1 - \exp\left(-3\frac{d}{d0}\right)\right] \quad (6)$$

where *g(d)* is the function describing the behavior of the variogram with respect to the distance *d, C0* is the nugget variance (i.e. the variance at the origin), *d0* is the decorrelation distance and *C1* is the value of the sill i.e. the sampling variance.

We do not present here a detailed description of the results obtained for the variograms calculated for the 30 selected situations but rather some general observations. For every studied situation and for almost all the distances the variogram obtained with a Bellon's correction is situated below the variogram obtained without any Bellon's correction as illustrated by Fig. 5 for one particular situation. It means that on average the distance *d* between the couples of VPRs is smaller when a Bellon’s correction is used. All the obtained variograms increase with the distance for distances between 5 km and 20 km. It means that for all the studied cases and for small distances between the VPRs close VPRs (geographically) are more similar than distant ones. We also observe that most of the variograms reach a sill for large distances (above 100 km) and that the values taken by this sill (from 15 dBZ^2 to 50 dBZ^2) are at least one order of magnitude higher than the maximum values of the variogram obtained for the theoretical case.

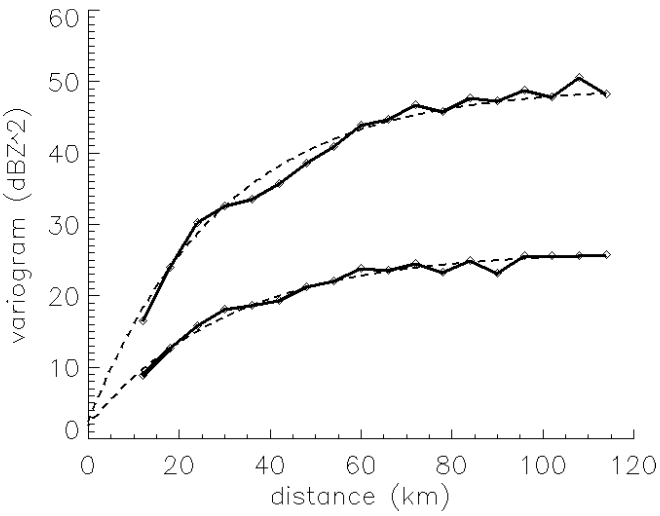

Fig. 5. Variogram obtained from volume reflectivity data on 29/07/2005 at 00:04 UTC with (lowest curve) and without (upper curve) a Bellon’s correction.

This suggests that the variations observed for the variograms obtained for the real cases mainly reflects the differences between couples of VPRs within a field of VPRs that is not spatially homogeneous rather than sampling errors caused by the measurement height effect and the beam broadening effect.

Finally the adjusted exponential model of variogram does not always fit the data. In particular for some cases the variogram increases up to a certain distance and then it decreases again instead of reaching a sill. A visual inspection reveals that the model (6) is visually good for about half of the tested cases.

5 Discussion and conclusion

Due to the limited number of elevation angles, the measurement height effect as well as the beam broadening effect (among other sources of error) the 'true' VPR is not directly accessible and can only be coarsely approximated. The study of one theoretical case reveals that due to these errors a uniform idealized VPR leads to a non zero variogram with a particular structure. The Bellon's correction applied to the apparent profiles gave variograms closer to zero for this theoretical case. A convergence to zero with an increasing number of elevation angles was also observed showing that the Bellon's procedure works properly. For 30 selected meteorological events the variograms obtained using the Bellon's correction were systematically lower than the variograms using no correction. The analysis of the variograms using a Bellon's correction showed that for all the studied cases close VPRs (geographically) are more similar than distant ones. We also observed that the variations of those variograms are not only explained by sampling errors due to the range effect but rather by the variations of VPRs within a non-uniform field. It means that using variograms to study the variability of the VPR does make sense. In only one half of the observed situations the variogram obtained agrees well with a selected exponential model. One must therefore be careful when giving an interpretation of the decorrelation distance derived from this model. The erratic behavior of the variogram obtained for about half of the situations may be due to the fact that the variogram is derived from a single volume scan. Future work will consist in studying the variogram averaged over several successive files over a longer time period.

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